

Mutual Information

- References:

- Cover, T. M., and J. A. Thomas, 1991: *Elements of Information Theory*. Wiley, 542 pp.
- DelSole, T., and M. K. Tippett, 2007: Predictability: Recent insights from information theory. *Rev. Geophys.*, 45, RG4002.

- Principle:

- Mutual information builds on concepts of information theory such as entropy, a measure of dispersion in a distribution. For a continuous probability distribution $p(x)$ for which $\int p(x)dx = 1$, the entropy across x is: $-\int p(x) \log p(x) dx$.
- Relative entropy, or Kullback-Leibler divergence, quantifies the difference between two probability distributions in x , $p(x)$ and $q(x)$ as: $\int p(x) \log[p(x)/q(x)] dx$, which is 0 if the two distributions are identical.
- Mutual information is a nonparametric measure of the dependence between two variables, similar to a correlation but without the assumption of linearity of the relationship. Stated another way, $I(x, y)$ is a measure of the reduction in the uncertainty of y due to knowledge of x .
- Consider x as values of a variable in a probability distribution – the distribution could be across space or time or both. Consider another variable y with its own distribution. Plotting y versus x yields a scatter diagram, and the Pearson's correlation coefficient is a measure of the fit of that scatter to a straight line. Mutual information is a goodness-of-fit independent of the assumption of a specific functional relationship, quantified as:

$$I(x, y) = \iint p(x, y) \log \left[\frac{p(x, y)}{p(x)p(y)} \right] dx dy$$

If x and y are independent, the value is 0.

- Data needs:

- Can be applied in the same situations as correlation – to relate two time series, two spatial distributions or space-time distributions.
- Suitable for non-continuous or incomplete data as well as complete data sets. However, the joint probability distribution $p(x, y)$ must have exactly the same data as the marginal probability distributions $p(x)$ and $p(y)$, i.e., no cells or times when one has missing data and the other does not.

- Caveats:

- For practical application to Earth system data, which is typically discrete in space (gridded or stations) and time (intervals or averages over intervals), integrals are replaced with summations.
- As with correlations, causality between variables is not assured, but more subtle and complex relationships than possible with correlation can be revealed.
- As noted above, all probability distributions must be normalized so that the sum of probabilities for each distribution = 1.
- For any bins where the distribution = 0, the term inside the summation should be set to 0.