On the Nature of Local Land-Atmosphere Coupling Strength for Vegetated Surfaces

"Little Omega"

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1 Evaporative Fraction and Soil Moisture Change

Transpiration by vegetation (LE_t), using the "Penman-Monteith" approach (Monteith, 1965), and the evaporative fraction for transpiration (ef_t) are

$$\begin{aligned} LE_t &= \frac{s(R_n - G) + \rho c_p g_a \delta e}{s + \gamma \left(1 + \frac{g_a}{g_c}\right)}, \\ ef_t &= \frac{s + \frac{\rho c_p g_a \delta e}{R_n - G}}{s + \gamma \left(1 + \frac{g_a}{g_c}\right)}, \end{aligned} \qquad (1)$$

where s is the slope of the saturation vapor pressure (with temperature), R_n is net radiation, G is soil heat flux, ρ is air density, c_p is specific heat of air, g_a is aerodynamic conductance (a measure of atmospheric turbulence), δe is the atmospheric vapor pressure deficit (a meausre of atmospheric humidity), γ is the psychrometric "constant", and g_c is canopy conductance. (Note that as $g_c \to \infty$, $LE_t \to LE_p$, the potential evaporation.) s and γ are

$$\begin{aligned} s &= \frac{de_s}{dT} = \frac{L_v}{R_v} \frac{e_s}{T^2}, \\ \gamma &= \frac{c_p p}{\epsilon L_v}, \end{aligned} \qquad (2)$$

where L_v is latent heat, R_v is the gas constant for water vapor, e_s is saturation vapor pressure, T is air temperature, p is surface air pressure, and ϵ is the ratio of the molecular weight of water vapor to dry air (0.622).

Soil heat flux is

$$G = \frac{\lambda_T (T_{sfc} - T_{ns})}{\delta z} = \frac{\lambda_T \delta T_{ns}}{\delta z}, \tag{3}$$

where λ_T is soil thermal conductivity, T_{sfc} and T_{ns} are the surface skin and near-surface soil temperatures, respectively (δT_{ns} is the near-surface soil temperature gradient), and δz is the nominal thickness of the near-surface soil layer (e.g. as in a land-surface model, LSM). Soil moisture (matric) potential (ψ, following Clapp and Hornberger, 1978, and Cosby et al 1984) and soil thermal conductivity (λ_T, following Al Nakshabandi and Kohnke, 1965) are, respectively

$$\begin{aligned} \psi &= \psi_{sat} \left(\frac{\Theta_{ns}}{\Theta_{sat}} \right)^{-\beta}, \\ \lambda_T &= a \exp[-b \ln(c\psi) + d], \end{aligned} \tag{4}$$

where ψ_{sat} is soil moisture potential at saturation, Θ_{ns} and Θ_{sat} are the near-surface and saturation (porosity) soil moisture values, respectively, and β is a coefficient, and $a = 23.9$, $b = \log(e)$, $c = 100$, and $d = 2.7$; ψ_{sat}, Θ_{sat} and β are functions of soil type. Alternate functions for soil moisture (matric) potential and soil thermal conductivity may be used, e.g. van Genuchten (1980), and Johansen (1975) as discussed in Peters-Lidard et al (1998), respectively.

Following Jarvis (1976, and others), canopy conductance, as expressed in many LSMs, may be written as

$$g_c = g_{s_{max}} LAI\, g_{s\downarrow} g_T g_{\delta e} g_\Theta, \tag{5}$$

where $g_{s_{max}}$ is maximum stomotal conductance, LAI is leaf area index (vegetation density), and $g_{s\downarrow}$, g_T, $g_{\delta e}$ and g_Θ are transpiration factors accounting for the effect incoming solar radiation, air temperature, atmospheric humidity deficit and soil moisture availability, respectively, all functions of vegetation type and environmental conditions. Soil moisture availability is defined as

$$g_\Theta = \frac{\Theta_{rz} - \Theta_{wilt}}{\Theta_{ref} - \Theta_{wilt}},$$

$$= \frac{\delta\Theta_{rz}}{\Theta_{ref} - \Theta_{wilt}}, \tag{6}$$

where Θ_{rz} is root zone soil moisture, Θ_{wilt} is soil moisture wilting point below which transpiration ceases, and Θ_{ref} is the soil moisture reference value above which transpiration not soil moisture limited ($\delta\Theta_{rz}$ is root zone volumetric soil moisture availability).

The changes in ψ, λ_T, G, and g_c with changing soil moisture are

$$\begin{aligned}
\frac{\partial\psi}{\partial\Theta} &= -\frac{\beta\psi}{\Theta_{ns}}, \\
\frac{\partial\lambda_T}{\partial\Theta} &= \frac{\partial}{\partial\Theta}\{a\exp[-b\ln(c\psi)+d]\} = -\frac{b\lambda_T}{\psi}\frac{\partial\psi}{\partial\Theta} = \frac{b\beta\lambda_T}{\Theta_{ns}}, \\
\frac{\partial G}{\partial\Theta} &= \frac{\delta T_{ns}}{\delta z}\frac{\partial\lambda_T}{\partial\Theta} = \frac{b\beta G}{\Theta_{ns}}, \\
\frac{\partial g_c}{\partial\Theta} &= \frac{g_c}{\delta\Theta_{rz}}.
\end{aligned} \tag{7}$$

Using (1) and (7), the change in *transpiration* fraction with changing soil moisture is then

$$\begin{aligned}
\frac{\partial ef_t}{\partial\Theta} &= \left(s + \frac{\rho c_p g_a \delta e}{R_n - G}\right)\frac{\partial}{\partial\Theta}\left\{\left[s + \gamma\left(1 + \frac{g_a}{g_c}\right)\right]^{-1}\right\} \\
&\quad + \frac{\rho c_p g_a \delta e}{s + \gamma(1 + \frac{g_a}{g_c})}\frac{\partial}{\partial\Theta}\left[(R_n - G)^{-1}\right], \\
&= \frac{s + \frac{\rho c_p g_a \delta e}{R_n - G}}{\left[s + \gamma\left(1 + \frac{g_a}{g_c}\right)\right]^2}\frac{\gamma g_a}{g_c^2}\frac{\partial g_c}{\partial\Theta} + \frac{\rho c_p g_a \delta e}{\left[s + \gamma\left(1 + \frac{g_a}{g_c}\right)\right]}\frac{1}{(R_n - G)^2}\frac{\partial G}{\partial\Theta}, \\
&= \frac{s + \frac{\rho c_p g_a \delta e}{R_n - G}}{\left[s + \gamma\left(1 + \frac{g_a}{g_c}\right)\right]^2}\frac{\gamma g_a}{\delta\Theta_{rz} g_c} + \frac{\rho c_p g_a \delta e}{\left[s + \gamma\left(1 + \frac{g_a}{g_c}\right)\right]}\frac{b\beta G}{\Theta_{ns}(R_n - G)^2}, \\
\frac{\partial \ln ef_t}{\partial\Theta} &= \frac{1}{\delta\Theta_{rz}}\left[\left(\frac{s + \gamma}{\gamma}\right)\frac{g_c}{g_a} + 1\right]^{-1} + \left[\frac{s(R_n - G)}{\rho c_p g_a \delta e} + 1\right]^{-1}\frac{b\beta}{\Theta_{ns}}\frac{G}{(R_n - G)},
\end{aligned}$$

$$= \frac{1}{\delta\Theta_{rz}} \left\{ \left[\left(\frac{s+\gamma}{\gamma} \right) \frac{g_c}{g_a} + 1 \right]^{-1} + \left[\frac{s(R_n - G)}{\rho c_p g_a \delta e} + 1 \right]^{-1} \frac{\delta\Theta_{rz}}{\Theta_{ns}} \frac{b\beta G}{(R_n - G)} \right\}. \quad (8)$$

Strictly speaking, (8) applies to the change in evaporative fraction with the change in *root zone* soil moisture, while the second term on the right hand side of (8) is with respect to *near-surface* soil moisture. But here we assume that $\Theta_{rz} \approx \Theta_{ns}$ so that (8) is still valid. Additionally, R_n contains the reflected solar radiation (a function of surface albedo) and emitted longwave radiation (a function of surface emissivity and surface skin temperature, T_s), and while these are affected by changes in soil moisture, we assume that the changes albedo and emissivity are small compared to the changes in canopy conductance and soil thermal conductivity with changing soil moisture, and that T_s is simply the balance of the components of the surface energy budget.

The relationship in (8) is described in Jacobs et al (2008) (although without the second term on the right hand side) and follows Jarvis and McNaughton (1986) who define a "decoupling" parameter (Ω) as

$$\Omega = \left[\left(\frac{\gamma}{s+\gamma} \right) \frac{g_a}{g_c} + 1 \right]^{-1}, \quad (9)$$

where $\Omega \to 0$ ($\Omega \to 1$) indicates strong (weak) land-atmosphere coupling. As an alternative, and intuitively appealling, we define a "coupling" paramter ω ($= 1 - \Omega$) from the first term on the right hand side of (8), where

$$\omega = \left[\left(\frac{s+\gamma}{\gamma} \right) \frac{g_c}{g_a} + 1 \right]^{-1}, \quad (10)$$

where $0 \leq \omega \leq 1$, so $\omega \to 1$ ($\omega \to 0$) indicates strong (weak) land-atmosphere coupling. Further, the second term on the right hand side of (8) is an additional coupling parameter defined as

$$\omega_G = \frac{\delta\Theta_{rz}}{\Theta_{ns}} \left[\frac{s(R_n - G)}{\rho c_p g_a \delta e} + 1 \right]^{-1} b\beta \frac{G}{(R_n - G)}, \quad (11)$$

where $0 \leq \omega_G <\approx O(1)$, so $\omega_G \gg 0$ ($\omega_G \to 0$) indicates strong (weak) land-atmosphere coupling. ω_G is typically much smaller than ω, and is included in the coupling parameter to account for "communication" between the soil and surface through the soil heat flux (G) –largest for wet soils (higher thermal conductivity) and weak turbulence and higher humidity– and also depends on atmospheric turbulence (g_a), humidity (δe), and the available energy ($R_n - G$).

Using (10) and (11), (8) may then be expressed simply as

$$\frac{\partial \ln ef_t}{\partial \Theta} = \frac{\omega + \omega_G}{\delta \Theta_{rz}}. \tag{12}$$

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3 References

Al Nakshabandi, G., and H. Kohnke, 1965: Thermal conductivity and diffusivity of soils as related to moisture tension and other physical properties. *Agric. Meteorol.*, **2**, 271-279.

Clapp, R. B., and G. M. Hornberger, 1978: Empirical equations for some soil hydraulic properties. *Water Resources Res.*, **14**, 601-604.

Cosby, B. J., G. M. Hornberger, R. B. Clapp, and T. R. Ginn, 1984: A statistical exploration of the relationship of soil moisture characteristics to the physical properties of soils. *Water Resources Res.*, **20**, 682-690.

Ek, M., and A. A. M. Holtslag, 2004: Influence of soil moisture on boundary-layer cloud development. *J. Hydrometeorol.*, **5**, 86-99.

Jacobs, C. M. J., E. J. Moors, H. W. Ter Maat, A. J. Teuling, G. Balsamo, K. Bergaoui, J. Ettema, M. Lange, B. J. J. M. van den Hurk, P. Viterbo, and W. Wergen, 2008: Evaluation of European Land Data Assimilation System (ELDAS) products using in situ observations. *Tellus*, **60**A(5), 1023-1037.

Jarvis P. G., 1976: The interpretation of the variations in leaf water potential and stomatal conductance found in canopies in the field. *Phil. Trans. R. Soc. Lond. B.*, **273**, 593-610.

Jarvis, P. G., and K. G. McNaughton, 1986: stomata1 control of transpiration: scaling up from leaf to region. *Adv. Ecol. Res.*, **15**, 1-49.

Johansen, O., 1975: Thermal conductivity of soils. Ph.D. thesis, University of Trondheim, 236 pp. [Available from Universitetsbiblioteket i Trondheim, Hgskoleringen 1, 7034 Trondheim, Norway.]

Mahrt, L., and H.-L. Pan, 1984: A two-layer model of soil hydrology. *Bound.-Layer Meteorol.*, **29**, 1-20.

Monteith, J. L., 1965: Evaporation and environment. *symp. soc. Exp. Biol.*, **19**, 205-234.

Pan, H.-L., and L. Mahrt, 1987: Interaction between soil hydrology and boundary-layer development. *Bound.-Layer Meteorol.*, **38**, 185-202.

Penman, H. L., 1948: Natural evaporation from open water, bare soil, and grass. *Proc. Roy. Soc. London*, **A193**, 120-146.

Peters-Lidard, C. D., E. Blackburn, X. Liang and E. F. Wood, 1998: The effect of soil thermal conductivity parameterization on surface energy fluxes and temperatures. *J. Atmos. Sci.*, **55** (7), 1209-1224.

van Genuchten, M. Th., 1980: A closed-form equation for predicting the hydraulic conductivity of unsaturated soils. *Soil Sci. Soc. Amer. J.*, **44**, 892-898.